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Evaluation of the Loss Coefficient in Mixtures Flows

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Abstract

This paper aims at demonstrating how we can obtain a dependency between pressure drop and other mixtures flows characteristic parameters (loss coefficient, dynamic viscosity) and we analyze this dependency numerically with concrete data from mixture transport in porous pipelines. The volume flow difference is generated by a corrosion crevice on pipe. The loss coefficient is calculated from a fraction of outlet flow and inlet flow. These results are represented on 2D and 3D graphics.

Key words *pressure, loss coefficient, flow, Reynolds number*

Introduction

In case of a fissure, the filter speed from a section depends on the value of the speed from the initial section and pressure drop is produced in the fissure. The result obtained has a starting point connected to a variation-law very general from the speeds of the filter along fissure.

Considering the case fissure, we will presuppose the filter speed on the wall tubes as being

$$v\big|_{r=R} = V(x)\,. \tag{1}$$

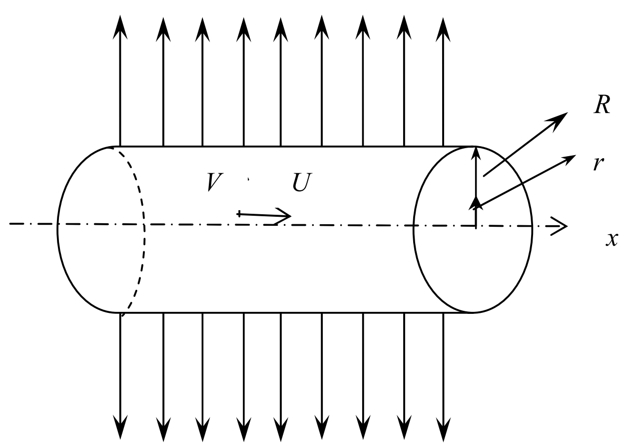

Fig. 1. The model of fluid flow in a tube

The move equation [1], in the system of selected coordinate as in Fig. 1 is:

$$\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial r}v=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right),$$

$$\frac{\partial v}{\partial x}u+\frac{\partial v}{\partial r}v=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right), \tag{2}$$

where we consider p pressure, with the υ cinematic fluid viscosity and have neglected fort of mass

$$u=0 \tag{3}$$

at $r=R$.

If we enter the dimensionless variable defined by the relation:

$$\overline{x}=\frac{x}{R},\ \overline{r}=\frac{r}{R} \tag{4}$$

and

$$\overline{u}=\frac{u}{u_0},\ \overline{v}=\frac{v}{u_0},\ \overline{V}=\frac{V}{u_0}, \tag{5}$$

respectively

$$\overline{p}=\frac{p}{\rho u_0^2}, \tag{6}$$

where u_0 is the value of reference speeds u, the equation becoming

$$\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial r}v=-\frac{\partial p}{\partial x}+\frac{1}{R_e}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right),$$

$$\frac{\partial v}{\partial x}u+\frac{\partial v}{\partial r}v=-\frac{\partial p}{\partial r}+\frac{1}{R_e}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right). \tag{7}$$

In view of simplification, we omit the bar and we use the notation

$$R_e=\frac{u_0 R}{\nu}. \tag{8}$$

The limit conditions for equation (7) have the form

$$u=0,\ v=V, \tag{9}$$

at $r=1$.

After eliminating pressure between equations (7), we obtain

$$\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}-\frac{v}{r}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial r}\right)=\frac{1}{R_e}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial r}\right) \tag{10}$$

and if we enter function ψ defined by relation:

$$u=\frac{1}{r}\frac{\partial}{\partial r}(r\psi),\ v=-\frac{\partial\psi}{\partial x}. \tag{11}$$

We discover this function equation:

$$\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial r}-\frac{v}{r}\right)\left(\frac{\partial^2}{\partial x^2}+D^2\right)\Psi=\frac{1}{R_e}\left(\frac{\partial^2}{\partial x^2}+D^2\right)\Psi\,, \tag{12}$$

where

$$D^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2}\,. \tag{13}$$

Function Ψ identically satisfies the equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0\,. \tag{14}$$

From (10) and (11) there results the condition to limit function Ψ:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\Psi)=0,\ \frac{\partial\Psi}{\partial x}=0 \tag{15}$$

at $r=1$.

Furthermore we enter function $\Phi(r,k)$, a transformation of Fourier from $\Psi(r,\,k)$:

$$\Phi(r,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{ikx}\Psi(x,r)\,\mathrm{d}\,x \tag{16}$$

and function $W(k)$ a transformation of Fourier from $V(x)$:

$$W(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{ikx}V(x)\,\mathrm{d}\,x\,. \tag{17}$$

Reverting to equation takes (12), we will consider the case in which Reynolds number is little, what permits us to neglect all terms from the left member.

If applying the Fourier transformation from the equation we obtain

$$(L^2-k^2)\Phi=0\,, \tag{18}$$

where

$$L^2=\frac{\mathrm{d}^2}{\mathrm{d}\,r^2}+\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}\,r}-\frac{1}{r^2}\,. \tag{19}$$

The limit condition becomes

$$\Phi(r,k)+r\frac{\partial\varphi(r,k)}{\partial r}=0,\ \ ik\Phi(r,k)=W(k) \tag{20}$$

at $r=1$.

From equation (18) we consider a solution of the form

$$\Phi(r,k)=AI_1(kr)+BrI_0(kr)\,. \tag{21}$$

After calculus, we find

$$A=-\frac{W(k)}{ik}\frac{2I_0(k)+kI_1(k)}{k\left[I_0^2(k)-I_1^2(k)\right]-2I_0(k)I_1(k)}\,, \tag{22}$$

$$B=\frac{W(k)}{ik}\frac{kI_0(k)}{k\left[I_0^2(k)-I_1^2(k)\right]-2I_0(k)I_1(k)}$$

and

$$\Phi(r,k) = -\frac{W(k)}{ik}\frac{\left[2I_0(k) + kI_1(k)\right]I_1(kr) - krI_0(k)I_0(kr)}{k\left[I_0^2(k) - I_1^2(k)\right] - 2I_0(k)I_1(k)}, \tag{23}$$

where

$$\Psi(x,r) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\Phi(r,k)\,\mathrm{d}k, \tag{24}$$

$$V(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}W(k)\,\mathrm{d}k. \tag{25}$$

The flow filter through the wall tubes ($r = 1$) [2], between sections $-x$ and x, he is

$$Q_f = 2\pi\left[\Psi(-x,-1) - \Psi(x,1)\right] = \sqrt{2\pi}\int_{-\infty}^{+\infty}(e^{ikx} - e^{-ikx})\Phi(1,k)\,\mathrm{d}k \tag{26}$$

of

$$Q_f = 2\sqrt{2\pi}\int_{-\infty}^{+\infty}\frac{W(k)}{k}\sin kx\cdot\mathrm{d}x. \tag{27}$$

The flow can write

$$Q_f = 4\pi x V(\xi), \tag{28}$$

with $\xi\in\left[-x,x\right]$, will have

$$xV(\xi) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{W(k)}{k}\sin kx\cdot\mathrm{d}k. \tag{29}$$

In indifferent transversal section tubes, the average speed has the expression

$$u_m(x) = 2\int_0^1 ur\cdot\mathrm{d}r. \tag{30}$$

From (11) and (24) we get

$$u_m(x) = \frac{2}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\Phi(1,k)\,\mathrm{d}k \tag{31}$$

and

$$u_m(-x) - u_m(x) = \frac{4}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{W(k)}{k}\sin kx\cdot\mathrm{d}k. \tag{32}$$

Comparing this formula with (29),

$$u_m(-x) - u_m(x) = 4xV(\xi) \tag{33}$$

or

$$u_m(0) - u_m(x) = 2xV(\xi), \tag{34}$$

with $\xi\in\left[0,x\right]$

The drop of pressure, on an elementary length $\mathrm{d}x$ tubes, has the expression:

$$\mathrm{d}p = -\frac{8}{R_e} u_m(x)\,\mathrm{d}x\,. \tag{35}$$

Introducing $u_m(x)$ from (34) and integral result:

$$p(0) - p(x) = \frac{8}{R_e}\left[u_m(0) - xV(\xi)\right]x\,. \tag{36}$$

We revert to the dimensional form

$$\Delta p = \frac{8\mu}{R^2}\left[u_m(0) - \frac{x}{R}V(\xi)\right]x\,. \tag{37}$$

Formerly we have noted with Δp the difference among the pressures $p(0)$ and $p(x)$.

Considering

$$\tau = \mu\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right). \tag{38}$$

Or, below a dimensionless form

$$\overline{\tau} = \frac{1}{R_e}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right), \tag{39}$$

where (4) and (5) are used.

On the wall tubes ($r=1$), obtained

$$\overline{\tau}_0 = \frac{1}{R_e}\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)\bigg|_{r=1} \tag{40}$$

and if using the previous result:

$$\frac{\partial u}{\partial r} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\frac{\partial^2\Phi(r,k)}{\partial r^2}\mathrm{d}k + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\frac{1}{r}\frac{\partial\Phi(r,k)}{\partial r}\mathrm{d}k - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\frac{\Phi(r,k)}{r^2}\mathrm{d}k\,, \tag{41}$$

from (23) results

$$\frac{\partial u}{\partial r}\bigg|_{r=1} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\frac{kW(k)}{i}\mathrm{d}k\,. \tag{42}$$

From (34) results

$$\frac{\partial v}{\partial x} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} k^2 e^{-ikx}\Phi(r,k)\,\mathrm{d}k \tag{43}$$

and using (23)

$$\frac{\partial v}{\partial x}\bigg|_{r=1} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\frac{kW(k)}{i}\mathrm{d}k\,. \tag{44}$$

Expression (40) for tangential efforts to the wall has the form:

$$\overline{\tau}_0 = \frac{2}{R_e\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-ikx}\frac{kW(k)}{i}\mathrm{d}k\,. \tag{45}$$

On an elementary length $\mathrm{d}x$ tubes, can be written as

$$\mathrm{d}p = 2\overline{\tau}_0 \,\mathrm{d}x \tag{46}$$

and after integration between sections $-x$ and x find

$$p(-x) - p(x) = \frac{4}{R_e\sqrt{2\pi}} \int_{-\infty}^{+\infty} (e^{ikx} - e^{-ikx}) W(k)\,\mathrm{d}k = \frac{4}{R_e}\left[V(-x) - V(x)\right] \tag{47}$$

using formula (25). Considering now sections 0 and x, we can write therefore:

$$p(0) - p(x) = \frac{4}{R_e}\left[V(0) - V(x)\right], \tag{48}$$

or after what we revert to dimensional measures

$$\Delta p = \frac{4\mu}{R_e}\left[V(0) - V(x)\right]. \tag{49}$$

Numerical Evaluation of Loss Coefficient

We presuppose stationary and slow flow, therefore R_e has small values and the loss coefficient α is constant.

From literature, we have:

$$\begin{aligned} Q_f &= 2\pi R \int_0^x V(x)\,\mathrm{d}x = 2\pi R V(0)x - 2\pi R\left[u_m(0)\frac{x}{R} - \frac{2}{3}V(\xi)\frac{x^2}{R^2}\right]x = \\ &= 2\pi R V(0)x - \frac{2}{3}\pi\left[u_m(0) + u_m(x)\right]x^2 \end{aligned}. \tag{50}$$

But

$$Q = Q_0 - Q_f = Q_0 - 2\pi R V(0)x + \frac{2}{3}\pi\left[u_m(0) + u_m(x)\right]x^2 \tag{51}$$

and results

$$\frac{Q}{Q_0} = 1 - \frac{2\pi R V(0)}{Q_0}x + \frac{2}{3Q_0}\pi\left[u_m(0) + u_m(x)\right]x^2. \tag{52}$$

From equation (52):

$$\frac{Q}{Q_0} = 1 - \frac{2\pi R V(0)}{\alpha Q_0}(1 - e^{-\alpha x}). \tag{53}$$

From $0 \le \alpha x \le \ln 2$ we can estimate with an error 3×10^{-3},

$$e^{-\alpha x} = 1 - 0{,}9664\alpha x + 0{,}3536\alpha^2 x^2. \tag{54}$$

In (53)

$$\frac{Q}{Q_0} = 1 - \frac{1{,}9328\cdot 2\pi R V(0)}{Q_0}x + \frac{0{,}7072\cdot 2\pi R V(0)}{Q_0}\alpha x \Rightarrow$$

$$\frac{Q}{Q_0} - 1 = \frac{2\pi R V(0)}{Q_0}x(0{,}7072\alpha - 1{,}9328).$$

After calculation,

$$\alpha = \frac{\dfrac{Q - Q_0}{2\pi R V_0 x} + 1{,}9328}{0{,}7072}. \tag{55}$$

In equation (55) α depending on the difference $\Delta Q = Q - Q_0$. If we consider de fraction of outlet flow $f = Q / Q_0$ (that must be proper fraction) and the known values for Q_0, V_0 and R it results a dependency of loss coefficient by this fraction:

$$\alpha(f) = \frac{\dfrac{Q_0 (f - 1)}{2\pi R V_0 x} + 1{,}9328}{0{,}7072}. \tag{56}$$

For a real value for mixture flows ($V_{oil} = V_0 = 100{,}8\,\mathrm{m}^3, V_{water} = 148\,\mathrm{m}^3$, $V_{gN} = 291 \cdot 10^3\,\mathrm{m}^3$ and $R = 8{,}4\,\mathrm{cm}$) can be derived $Q_0 = 3{,}371\,\mathrm{m}^3/\mathrm{s}$ and $V_0 = 151{,}57\,\mathrm{m/s}$. For these values the linear dependency is represented in Fig. 2.

According to equation (49) the pressure drop Δp depends on the dynamic viscosity and loss coefficient α. These dependences for $\alpha_1 = 0{,}001\,\mathrm{m}^{-1}$, $\alpha_2 = 0{,}002\,\mathrm{m}^{-1}$ and $\alpha_3 = 2{,}733\,\mathrm{m}^{-1}$ are illustrated on Fig. 3.

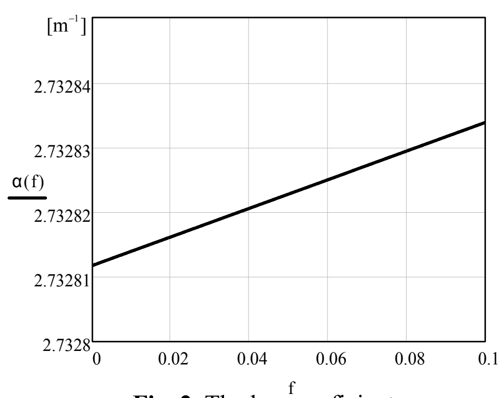

Fig. 2. The loss coeficient dependence by the fraction of outlet flow

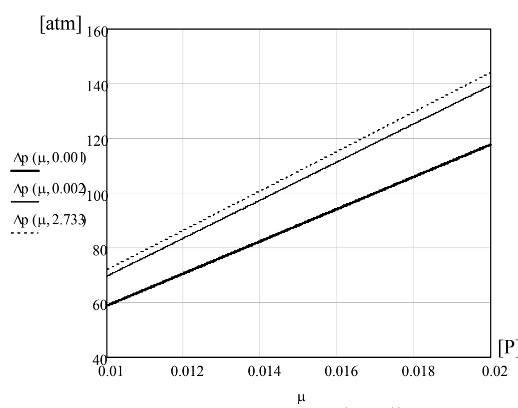

Fig. 3. Pressure drop linear dependency for several values of the fraction of outlet flow

Using equation (46) the pressure drop is a function with two variables:

$$\Delta p(\mu, \alpha) = \frac{4\mu}{R_e}\left(1 - \mathrm{e}^{-\alpha x}\right) V_0 \tag{57}$$

and this function can be represented as 3D graphics as surface plot (Fig. 4) or contour plot (Fig. 5). In this case we have $V_0 = 151{,}57\,\mathrm{m/s}$, $R_e = 8{,}4\,\mathrm{cm}$ and $x = 1{,}7\,\mathrm{km}$.

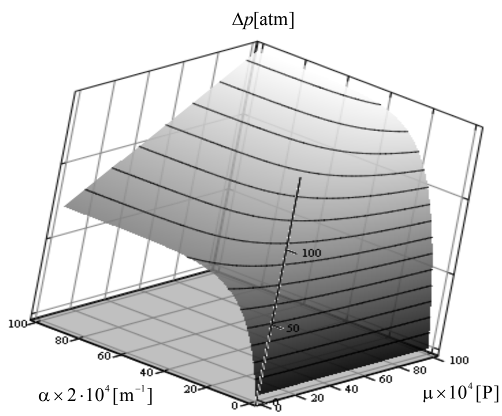

Fig. 4. The loss coeficient dependence by the fraction of outlet flow

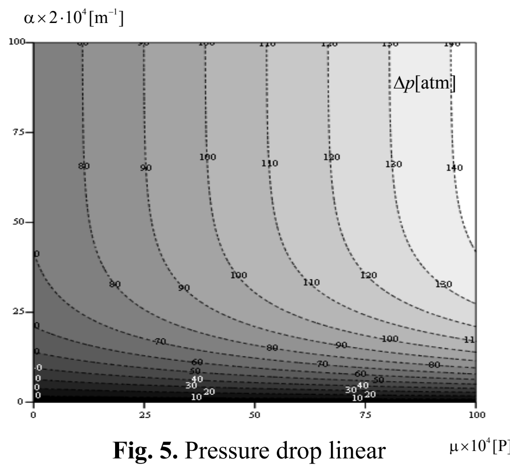

Fig. 5. Pressure drop linear dependency for several values of the fraction of outlet flow

Conclusions

As shown, the pressure drop Δp depends not only on the loss coefficient α but also on the dynamic viscosity μ. In Fig. 4 we can observe that the pressure drop dependency is more relevant in case of large values of loss coefficient. To minimize the pressure drop any of these variables must be decreased, but it is more efficient to minimize the loss coefficient.

References

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Evaluarea coeficientului de pierdere în curgerea bifazică

Rezumat

În această lucrare se calculează componentele tensorilor permitivităţii gravitaţionale relative şi permeabilităţii gravitaţionale relative ai vacuumului modificat de o undă gravitaţională completă. Tensorii permitivităţii şi permeabilităţii gravitaţionale corespunzători sunt nediagonali şi sunt diferiţi în aproximaţia de ordinul al doilea al puterilor amplitudinilor. Se analizează cazurile particulare semnificative şi valorile mediate în spaţiu şi timp.